MHD FLOW AND HEAT TRANSFER THROUGH A POROUS MEDIUM OVER A STRETCHING/SHRINKING SURFACE WITH SUCTION

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ABSTRACT: *This article examines MHD flow and heat transfer through a porous medium over a shrinking surface with suction. The governing partial differential equations of fluid motion are converted into ordinary differential form by using similarity functions. The resulting equations have been solved numerically to observe the effects of physical parameters of this study namely magnetic parameter* M^2, *permeability paramete* K, *suction parameter* λ *and Prandt number* $\Pr$. *Results are presented in graphical form.*

AMS Subject Classification: 76M20.
Key Words: MHD flow, Heat transfer, Porous medium, Prandtl number, Suction, Shrinking / Stretching surface.

1. INTRODUCTION

The flow and heat transfer over a stretching surface bears important research interest due to its various applications in industries such as hot rolling, wire drawing, glass fiber production, manufacturing plastic films and extrusion of a polymer in a melt spinning process. Sakiadis [23, 24] was the first to propose and analyze the surface stretching problem based on the boundary layer approximation. Crane [9] gave a closed-form solution for the steady two-dimensional flow of an incompressible viscous fluid caused by the stretching of an elastic sheet, which moves in its own plane with a velocity that varies linearly with distance from a fixed point. Gupta and Gupta [13] extended the work of Crane [9] by investigating the effect of mass transfer on a stretching sheet with suction or blowing for linear surface velocity subject to a uniform temperature. Magyari and Keller [17] introduced a new type of the stretching sheet problem by considering the flow due to a sheet stretched exponentially in its own plane, and they investigated also the heat transfer characteristics for the flow taking the exponentially varying wall temperature. Barik et al. [7] studied heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat source.

Miklavcic and Wang [18] obtained the solution for steady two-dimensional, as well as axisymmetric viscous, flow over a shrinking sheet. Wang [25] studied two dimensional stagnation flow towards a shrinking sheet. Muhaimin et al. [19] observed the effects of heat and mass transfer on MHD boundary layer flow past a shrinking sheet subject to suction. Cortell [10] discussed MHD viscous flow caused by a shrinking sheet with suction for two-dimensional and axisymmetric cases. Fang and Zhang [12] obtained the closed form solution for steady MHD flow over a shrinking surface subject to applied suction. Fang [11] investigated the flow over a shrinking sheet to power law surface velocity and obtained the multiple solutions for certain mass transfer with controlling parameters. Noor et al. [20] analyzed the magnetohydrodynamic viscous flow due to a shrinking sheet analytically and found that the result obtained by a domian decomposition and homotopy analysis methods are well agreed. Sajid and Hayat [22] investigated the effect of the MHD for two-dimensional and axisymmetric shrinking sheet. Bhattacharyya and Pop [8] investigated MHD boundary layer flow due to an exponentially shrinking sheet. Asghar et al. [5] presented an exact analytical solution for the boundary layer flow of a viscous fluid over an impermeable shrinking sheet. Hayat et al. [14, 15] reported an analytic HAM solution for the MHD flow of a second grade fluid over a shrinking sheet without and with rotation effects.

In recent years, the study of magnetohydrodynamic (MHD) flow problems has gained considerable interest of researchers because of its huge applications in many engineering problems. Pavlov [21] discussed the MHD boundary layer flow of an electrically conducting fluid due to the stretching sheet. Whereas, Ali et al. [5] studied heat transfer on magnatohydrodynamic (MHD) viscous flow over stretching sheet with prescribed heat flux and Hussain et al. [16] investigated MHD stagnation point flow of micropolar fluids towards a stretching sheet. Baag et al. [6] analyzed MHD flow on a stretching sheet embedded in a porous Medium. Flow of an electrically conducting non- Newtonian fluid past a stretching surface was studied by Able et al. [1] when a uniform magnetic field acts transverse to the surface. Ahmad and Sajjad [2] obtained exact solution for a viscous, incompressible, MHD flow over a porous stretching sheet. Ali et al. [3] presented numerical solution of MHD flow of fluid and heat transfer over porous stretching sheet.

2. MATHEMATICAL ANALYSIS

Consider steady, two dimensional and incompressible laminar flow of viscous fluid through porous medium over a stretching/ shrinking surface. The fluid is electrically conducting. Magnetic field of strength B_0 is applied in normal direction to the sheet. A convective heat source with heat flux boundary conditions provides temperature T_w at the surface. The Cartesian coordinates are used. The x-axis is along the sheet and y-axis is perpendicular to it. The origin is fixed. Here u, v are velocity components along horizontal and vertical directions. The induced magnetic field is neglected. The permeability of medium is K_0. The governing equations of the motion are:

$$\partial u/\partial x+\partial v/\partial y=0, \quad (1)$$

$$u\partial u/\partial x+v\partial u/\partial y=\upsilon\partial^2 u/\partial y^2-\frac{\sigma B_0^2 u}{\rho}-\frac{\upsilon}{K_0}u, \quad (2)$$

$$u\partial T/\partial x+v\partial T/\partial y=\frac{K'}{\rho C_p}\partial^2 T/\partial y^2 \quad (3)$$

where μ is dynamic viscosity, ρ is fluid density, υ kinematic viscosity, C_p is the specific heat at constant pressure and K' is thermal conductivity.

The associated boundary conditions are:

$$u=\pm ax, \quad \upsilon=-v_0, \quad -K'\frac{\partial T}{\partial y}=q_w=E_0x^2 \text{ at y = 0} \quad (4)$$

$$u\to 0,\ T\to T_w \text{ as y}\to\infty \quad (5)$$

wher $u>0$ corresponds with flow over stretching surface and $u<0$ for flow over shrinking surface.

The similarity transformations are as follows:

$$u=cxf'(\eta), v=-\sqrt{\upsilon c}\,f(\eta) \quad (6)$$

$$T-T_\infty=\frac{E_0x^2}{K'}\sqrt{\frac{\upsilon}{a}}\theta(\eta) \quad (7)$$

where $\eta=y\sqrt{\frac{a}{\upsilon}}$ is dimensionless variable, E_0 is positive constant, T_∞ is temperature far away from the surface, q_w is ratio of heat transfer.

The equation (1) is readily satisfied and the equations (2) and (3) become:

$$f'''+ff''-f'^2-(M^2+\frac{1}{K})f'=0, \quad (8)$$

$$\theta''+\Pr f\theta'-2\Pr f'\theta=0, \quad (9)$$

where prime denotes the differentiation with respect to η, $M^2=\frac{\sigma B_0^2}{\rho c}$ is magnetic parameter, $K=\frac{K_0 c}{\upsilon}$ is permeability parameter and $\Pr=\frac{\mu C_p}{K'}$.

The boundary conditions (4) and (5) then become:

$$\left.\begin{array}{l} f(0)=\lambda,\ f'(0)=\pm 1,\ \theta'(0)=-1, \\ f'(\infty)=0, \theta(\infty)=0. \end{array}\right\} \quad (10)$$

3. RESULTS AND DISCUSSION

The equations (8) and (9) are solved subject to the boundary conditions (10). The numerical results have been computed by using Mathematica 6.0 software. The effects of the physical parameters namely magnetic parametere M^2, suction parameter λ, permeability parametere K and Prandtl number $\Pr$ have been noticed on velocity and temperature distributions. The results have been presented graphically. The fluid flow due to stretching surface is described in fig-1 to fig-3. Fig-1 and Fig-2 respectively show that the velocity component f' decreases with increasing values of M^2 and λ. Fig-3 demonstrates that velocity f' increases with increase in the value of K. The fluid flow over a shrinking surface is described in Fig-4 to Fig-6. The Fig-4, Fig-5 and Fig.6 respectively show that the velocity f' increases with increasing values of M^2, λ and K.

The temperature distributions for flow due to stretching surface are presented in Fig-7 and Fig-8. The temperature function $\theta(\eta)$ decreases with increasing values of $\Pr$ and λ. Fig-9 and Fig-10 respectively demonstrate the effects of $\Pr$ and λ on $\theta(\eta)$ for flow over shrinking surface. The temperature function $\theta(\eta)$ decreases for increase in the values of $\Pr$ and λ. Fig-11 shows that temperature function $\theta(\eta)$ is higher for flow over shrinking surface than for stretching surface.

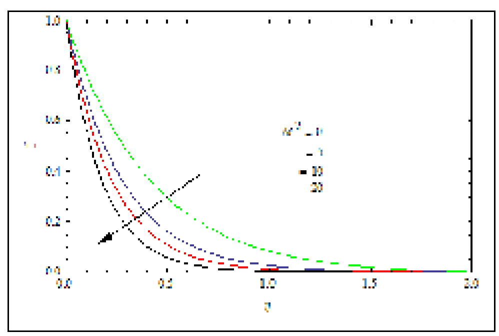

Fig-1 Graph of f' for different values of M^2 when K=100, λ=2, Pr=0.1

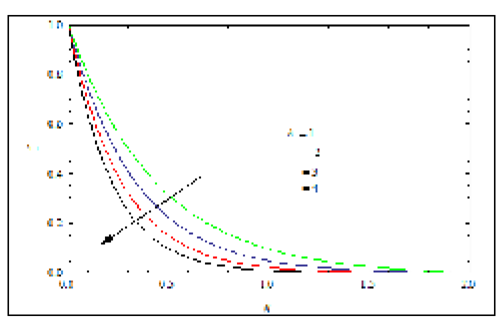

Fig-2 Graph of f' for different values of λ, when K=100, M^2=2, Pr=0.1

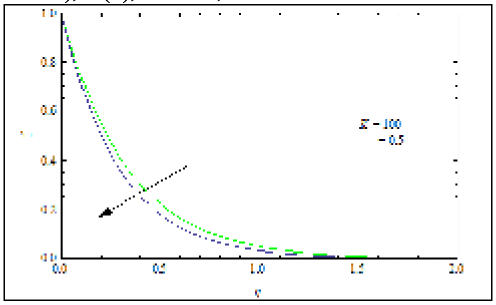

Fig-3 Graph of f′ for different values of K when M^2=100, λ=2, Pr=0.1

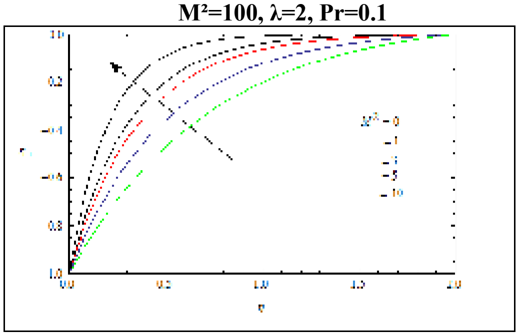

Fig-4 Graph of f′ for different values of M^2 when K=100, λ=2, Pr=0.1

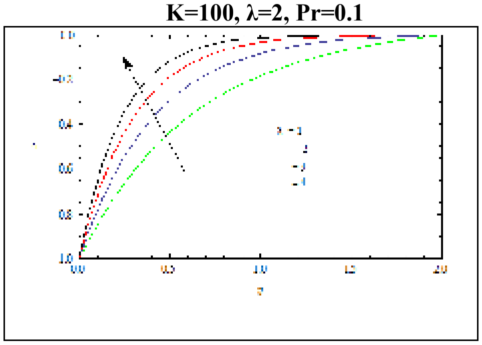

Fig-5 Graph of f′ for different values of λ when K=100, M^2=2, Pr=0.1

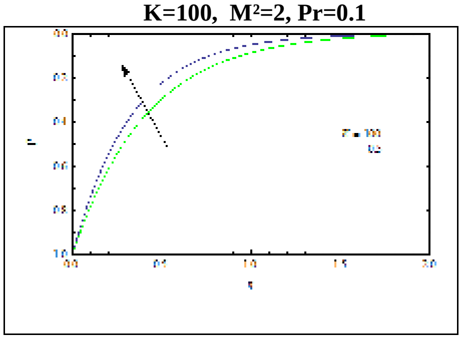

Fig. 6 Graph of f′ for different values of M^2 when K=100, λ=2, Pr=0.1

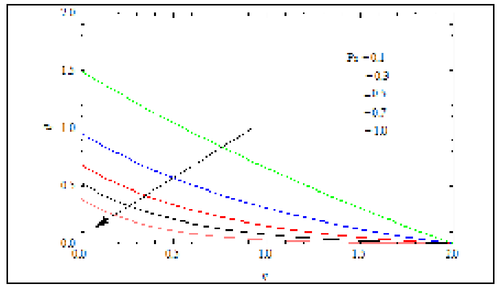

Fig-7 Graph of θ for different values of Pr when K=100, λ=2, M^2=2

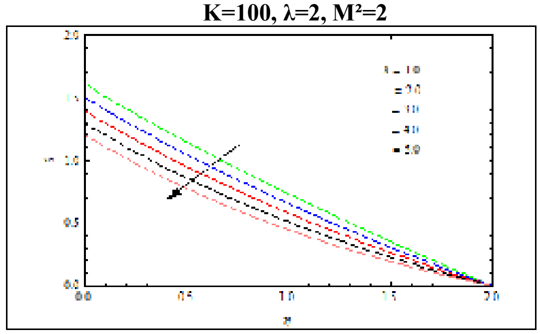

Fig-8 Graph of θ for different values of λ when K=100, M^2=2, Pr=0.1

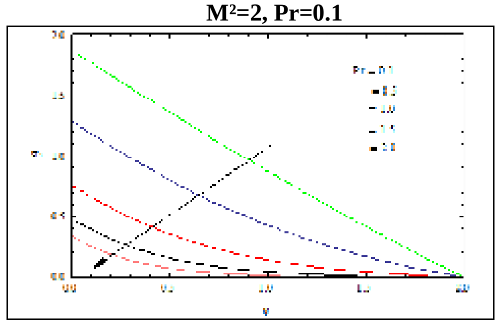

Fig-9 Graph of f′ for different values of Pr when K=100, λ=2, M^2=2

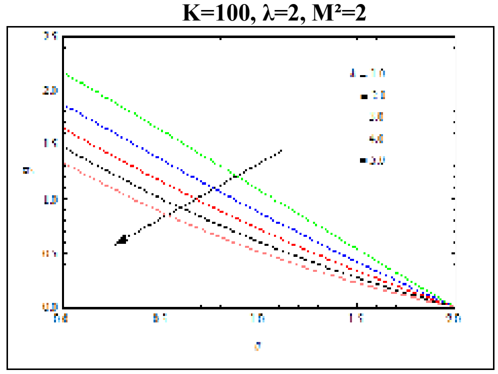

Fig-10 Graph of θ for different values of λ when K=100, M^2=2, Pr=0.1

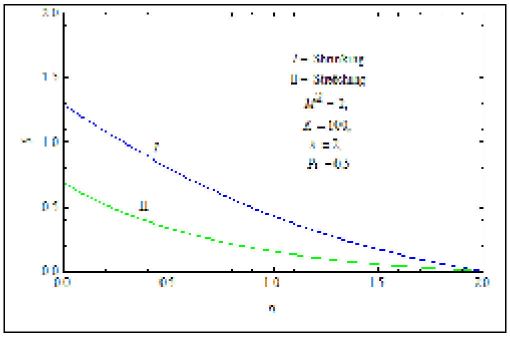

Fig-11 Graph for the comparison of the values of θ for flow over stretching/shrinking surface

4. CONCLUSION

Numerical solution is obtained for MHD flow over a stretching / shrinking surface with suction and heat transfer. The main findings of this study are summarized as follows:

- The velocity component f' for fluid flow over stretching surface decreases with increasing values of M^2 and λ but reverse effects have been noticed for flow over shrinking surface.
- The temperature function $\theta(\eta)$ decreases with increasing values of $\Pr$ and λ. This result hold for fluid flow over stretching/shrinking surface but the temperature distribution is higher for flow over shrinking surface than for stretching surface.
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REFERENCES

1. S. Able, P. H. Veena, K. Rajagopal & V. K. Pravin, Non- Newtonian magnetohyderodynamic flow over a stretching surface with heat and mass transfer. International Journal of Nonlinear Mechanics 39, 1067-1078, 2004.
2. F. Ahmad, S. Hussain and M. Ali, An Analytical Solution of MHD Flow over Porous Stretching Sheet, J. Basic. Appl. Sci. Res., 4(3)160-167, 2014.
3. M. Ali, F. Ahmad and S. Hussain, Numerical Solution of MHD Flow of Fluid and Heat Transfer over Porous Stretching Sheet, J. Basic. Appl. Sci. Res., 4(5)146-152, 2014.
4. M. Ali, F. Ahmad, S. Hussain, Heat Transfer On Magnatohydrodynamic (MHD) Viscous Flow Over Stretching Sheet With Prescribed Heat Flux, *J. Basic Appl. Sci.* 4(7)28-35, 2014.
5. S. Asghar, A. Ahmad and A. Alsaedi, Flow of a viscous fluid over an incompressible shrinking sheet, Applied Mathematics Letters, 26(12),1165-1168, 2013.
6. S. Baag, M. R. Acharya, G. C. Dash, MHD Flow Analysis Using DTM-Pade' and Numerical Methods, American Journal of Fluid Dynamics, 4(1), 6-15, 2014.
7. R. N. Barik, G. C. Dashand & P. K. Rath, Heat and mass transfer on MHD flow through a porous Medium over a stretching surface with heat source,Mathematical Theory and Modeling , 2(7), 49-60, 2012.
8. K. Bhattacharyya and I. Pop, MHD boundary layer flow due to an exponentially shrinking sheet, magnetohydrodynamics, 47(4), 337--344, 2011.
9. L. J. Crane, Flow past a stretching plate, Zeitschrift f¨ ur Angewandte Mathematik und Physik, 21(4), 645--647, 1970.
10. R. Cortell. On a certain boundary value problem arising in shrinking sheet flows. Appl. Math. Comput., 217, 4086--4093, 2010.
11. T. G. Fang, Boundary layer flow over a shrinking sheet with power-law velocity, International Journal of Heat and Mass Transfer, 51(25-26), 5838--5843, 2008.
12. T. Fang and J. Zhang, Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Communications in Nonlinear Science and Numerical Simulation, vol. 14, no. 7, pp. 2853--2857, 2009.
13. P. S. Gupta and A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, The Canadian Journal of Chemical Engineering, 55 (6), 744--746, 1977.
14. T. Hayat, Z. Abbas, M. Sajid. On the analytic solution of magnetohydrodynamic flow of a second grade fluid over a shrinking sheet. ASME J. Appl. Mech., 74, 1165--1171, 2007.
15. T. Hayat, T. Javed, M. Sajid. Analytic solution for MHD rotating flow of a second grade fluid over a shrinking surface. Phys. Lett. A, 372, 3264--3273, 2008.
16. S. Hussain, M. A. Kamal,, F. Ahmad, and M. Shafique, MHD stagnation point flow of micropolar fluids towards a stretching sheet, Sci.Int.(Lahore), 26(5), 1921-1929, 2014.
17. E. Magyari, B. Keller. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. J. Phys. D. Appl. Phys., 32, 577--585, 1999.
18. M. Miklavcic and C. Y. Wang, Viscous flow due to a shrinking sheet, Quarterly of Applied Mathematics, 64(2), 283--290, 2006.
19. I. Muhaimin, R. Kandasamy and A.B. Khamis, Effects of heat and mass transferon non-linearMHD boundary layer flow over a shrinking sheet in the presence of suction, Applied Mathematics and Mechanics, 29(10),1309-1317, 2008.
20. N. F. M. Noor, S. A. Kechil, I. Hashim. Simple non-perturbative solution for MHD viscous flow due to a shrinking sheet. Commun. Nonlinear Sci. Numer. Simul., 15, 144--148, 2010.
21. K. B. Pavlov. Magnetohydrodynamic flow of an incompressible viscous fluid caused by the deformation of a plane surface. Magnetohydrodynamics, 10(4), 507--509, 1974.
22. M. Sajid and T. Hayat, The application of homotopy analysis method for MHD viscous flow due to a shrinking sheet, Chaos, Solitons and Fractals, 39(3), 1317--1323, 2009.

23. B. C. Sakiadis, Boundary layer behavior on continuous solid surface: I. Boundary layer equations fortwo dimensional and axisymmetric flow, AIChE Journal, 7(1), 26--28, 1961.
24. B. C. Sakiadis, Boundary layer behavior on continuous solid surface: II. Boundary layer equations for two dimensional and axisymmetric flow, AIChE Journal, 7(1), 221--225, 1961.
25. C. Y. Wang, Stagnation flow towards a shrinking sheet, Int. J. Non- Linear Mechanics, 43, 377-382, 2008.